Massachusetts Institute of Technology

Navigating the String Landscape with Machine Learning Techniques

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IAIFI Colloquium, Sept 2024

<u>In collaboration with</u>

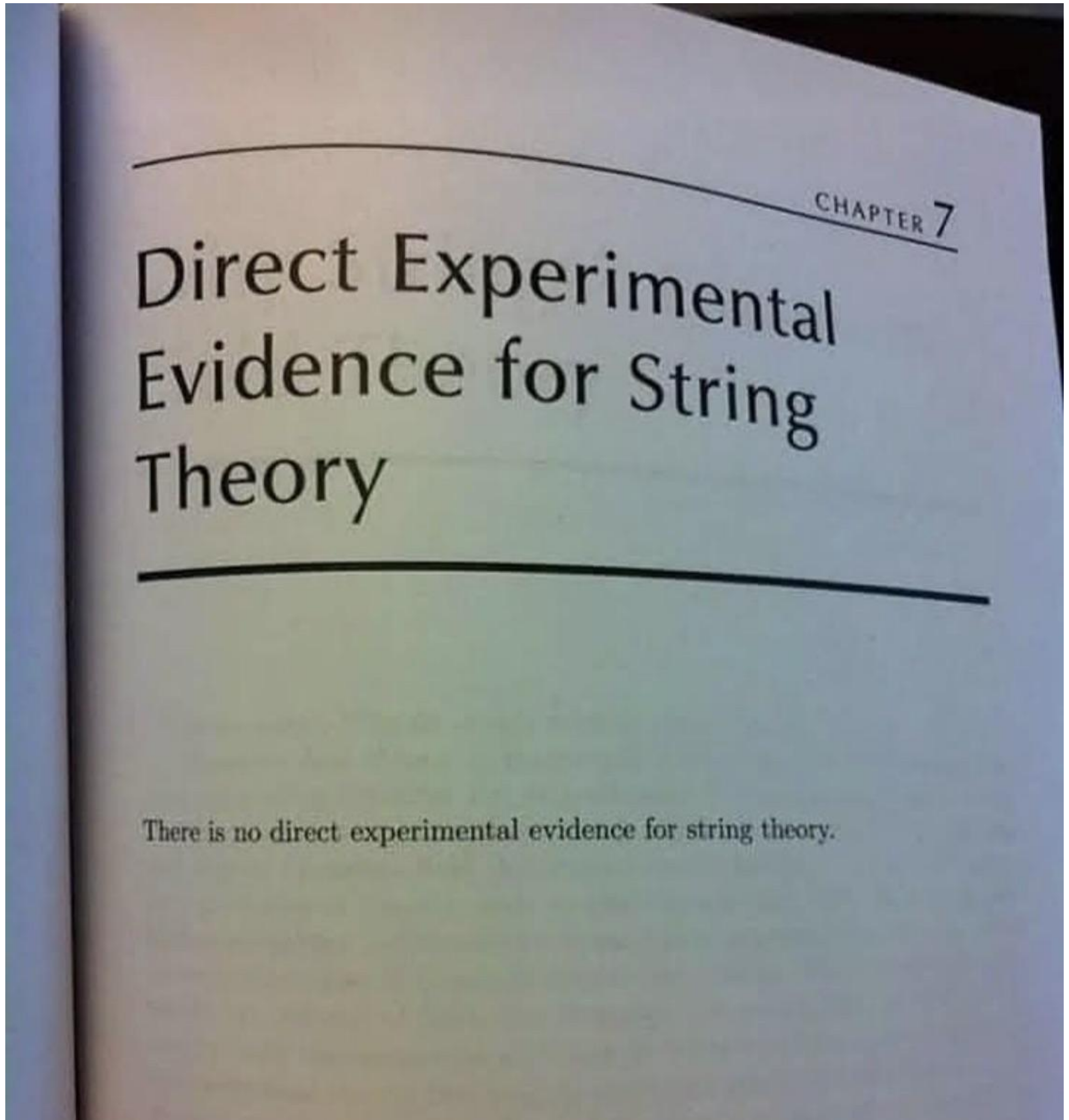
Steve Abel[a], Andrei Constantin[b], Cristofero Fraser-Taliente[c], Andre Lukas[d], Burt Ovrut[e]

<u>Based on</u>

2108.07316[bd], 2110.14029[abd], 2111.07333[abd], 2402.01615[bcde] , 2410.xxxx[bcde]

Why String Theory?

- On googling “why string theory?”, you may find the following image



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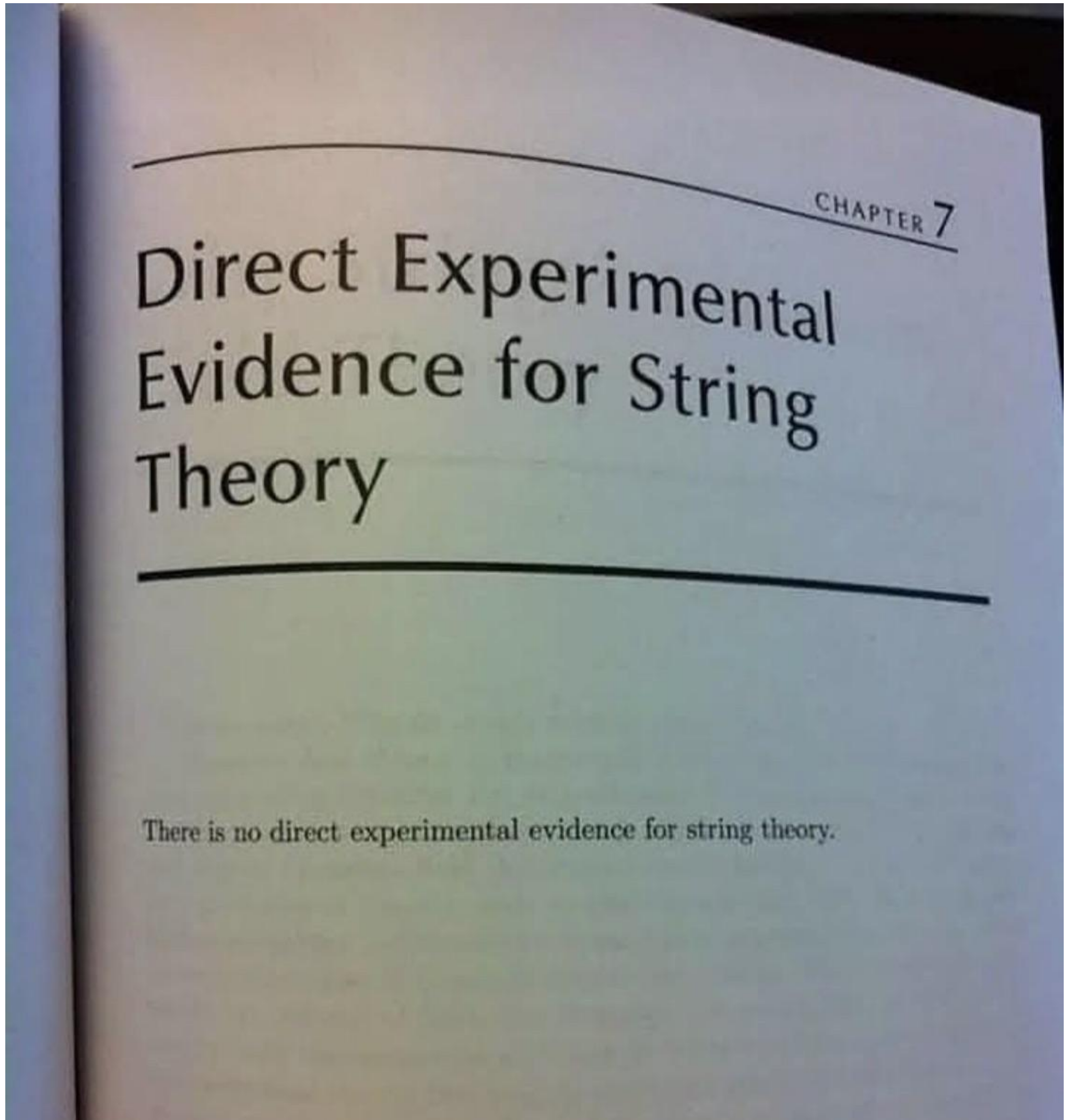

This book is by Joe Conlon. What are his recent publications?

A Note on 4d Kination and Higher-Dimensional Uplifts
Fien Apers (Oxford U., Theor. Phys.), Joseph P. Conlon (Oxford U., Theor. Phys.), Marti
(Sep 12, 2024)
e-Print: 2409.08049 [hep-th]
pdf cite claim

Percolating Cosmic String Networks from Kination
Joseph P. Conlon (Oxford U., Theor. Phys.), Edmund J. Copeland (Nottingham U.), Edw
Noelia Sánchez González (Oxford U., Theor. Phys.) (Jun 18, 2024)
e-Print: 2406.12637 [hep-ph]
pdf cite claim

String Theory and the Early Universe: Constraints and Opportunities
Joseph P. Conlon (Oxford U., Theor. Phys.) (May 29, 2024)
Contribution to: Moriond Cosmology 2024 • e-Print: 2405.19118 [astro-ph.CO]
pdf cite claim

Out of the dark: WISPs in String Theory and the Early Universe
Joseph P. Conlon (Feb 1, 2024)
Published in: *PoS* COSMICWISPers (2024) 001 • Contribution to: COSMICWISPers, 00
pdf DOI cite claim

String theory and the first half of the universe
Fien Apers (Oxford U., Theor. Phys.), Joseph P. Conlon (Oxford U., Theor. Phys.), Edmu
Martin Mosny (Oxford U., Theor. Phys.), Filippo Revello (Utrecht U.) (Jan 8, 2024)
Published in: *JCAP* 08 (2024) 018 • e-Print: 2401.04064 [hep-th]
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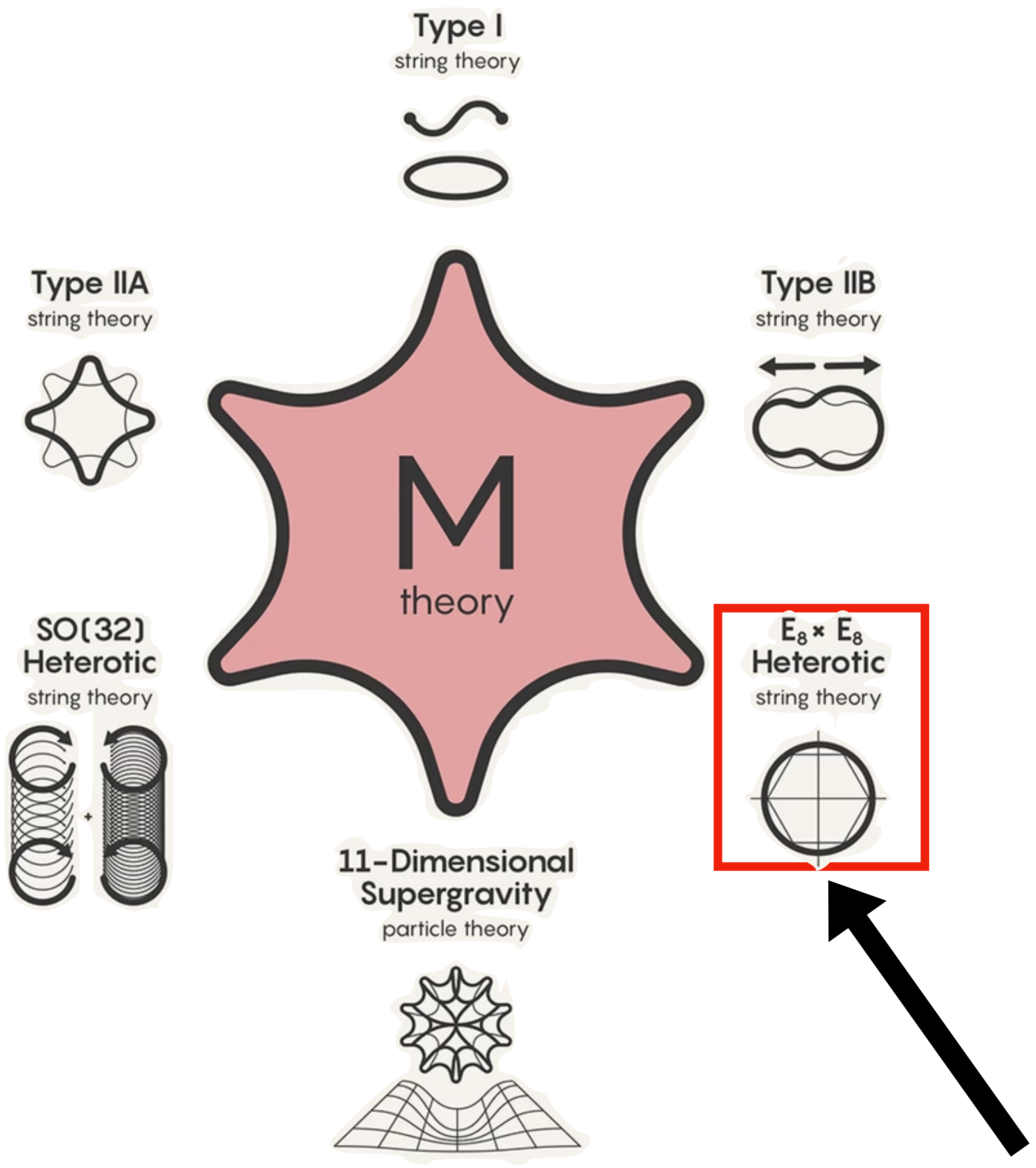
Why String Theory?

- String Dualities & holography have allowed calculations of strongly coupled theories
- Microstate counting for black hole entropy -> matches Hawking
- Intricate connections to Mathematics - e.g. Mirror symmetry
- String theory IS a theory of quantum gravity - is it the right one?
- It contains non-abelian gauge theories and chirality - the basis of particle physics
- At low energies, and large scales, the gravity theory is Einstein gravity

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{1}{M_{pl}^2}T_{\mu\nu} + \mathcal{O}\left(\frac{E^2}{M_{pl}^4}\right)$$

String Theory

- Particles are replaced with various extended objects called branes.
- There are five different limits of string theory the theory, where the universe is perturbatively described by one dimensional objects called strings in 10 dimensions
- In other limits the universe appears to be well described by 11D supergravity



The focus of this talk

String Theory

- The obvious problem: String theory exists in 10 (11) dimensions
- We need initial conditions (i.e. We need to specify a 10D geometry)
- Compactifications:

$$M_{10} = \mathbb{R}^{1,3} \times M_6 \xrightarrow{V_6 \to 0} \mathbb{R}^{1,3} \qquad S = \int_{M_{10}} d^{10}x \mathscr{L} \approx V_6 \int_{\mathbb{R}^{1,3}} d^4x \left[\mathscr{L}^{\Lambda}_{eff} + \mathcal{O}\left(\Lambda V_6^{1/6}\right)\right]$$

- Different choices of M_6, and field profiles of it, lead to different 4D Physics $\mathscr{L}^{\Lambda}_{eff}$
- We will make use of supersymmetry - this is not to say we have low energy SUSY
- For our purposes, this will mean that M_6 is a Calabi-Yau manifold (CY 3-fold)

String Theory

- The obvious problem: String theory exists in 10 (11) dimensions
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- Compactifications:

<u>General rule</u>
(Quasi-)Topological ~ Particle Spectrum (First part of talk)
Geometric ~ Coupling constants (Second part of talk)

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The recipe for a string compactification

- Aim: Particle Spectrum, Yukawa couplings, and Stabilise Moduli
- For the heterotic string, we have gauge fields A charged under E_8 X E_8
- We need a Calabi-Yau manifold for N=1 SUSY in 4D
- We need a vector bundle V over the Calabi-Yau for N=1 SUSY in 4D
  - The structure group of this vector bundle determines the low energy gauge group
  - $E_8 \supset SU(5) \times \underline{SU(5)}, SU(4) \times \underline{SO(10)}, SU(3) \times \underline{E_6}$

These are the standard GUT Groups
Broken the SM by appropriate Wilson line

The recipe for a string compactification

<u>Topology (Discrete Data)</u>

CY Manifold

Gauge Bundle

Wilson Line

↓

Low Energy Massless Spectrum
(Discrete Data of theory)
(Cohomology)

The recipe for a string compactification

Topology (Discrete Data)

CY Manifold

Gauge Bundle

Wilson Line

Equations of Motion
(PDEs in 6D)

Unique Solution

1. Yau's Theorem
2. Donaldson-Uhlenbeck-Yau Theorem

Geometry (Solutions to PDEs)

Metric $g_{\mu\nu}$

Gauge Connection A^{μ}

Low Energy Massless Spectrum
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The recipe for a string compactification

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Geometry (Solutions to PDEs)

Metric $g_{\mu\nu}$

Gauge Connection A^{μ}

Low Energy Massless Spectrum
(Discrete Data of theory)
(Cohomology)

Low Energy Couplings
(Continuous Data of Theory)

Why Machine Learning and String Theory?

TLDR: String Theory has big data

Number of (known) Calabi-Yau Manifolds ~ 10^{400} (Chandra et al 2023 & Gender et al 2023)

Approximate Number of Perturbative Flux vacua in IIB ~ 10^{500} (Ashok and Douglas 2004)

Extension to Strong Coupling ~ $10^{272,000}$ (Taylor and Wang 2015)

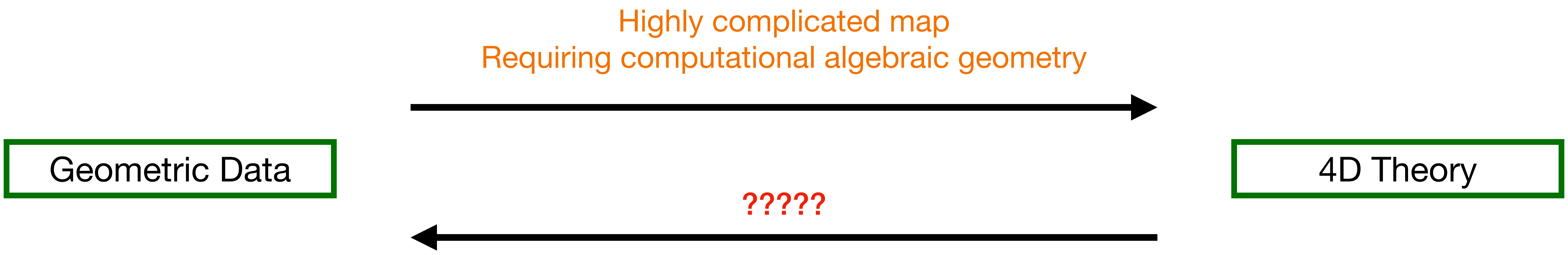
This data is also clean!

Pure mathematics with zero noise

Why Machine Learning and String Theory?

TLDR: String Theory has an inverse problem

- Focus on spectrum for now
- To work out particle spectrum in 4D need (integer) topological data about the manifold and field profiles of it



- Incredibly hard to identify the few constructions of phenomenological interest!

Why Machine Learning and String Theory?

Large Datasets: c.f. Previous slide - conjecture generation

Discrete Optimisation: What compactifications lead to the standard model?

Solving PDEs: Solving 6D Einstein equations, CY metrics and field profiles.
Allows calculation of Yukawa couplings

Geometrically Engineering the SM with RL

Building Bundles on Calabi-Yau Manifolds

- What Geometric Data?
  - Fix a given CY manifold
  - Monad Bundles - Non-abelian bundles formed from line bundles
- A line bundle are specified by its first Chern class - a vector of integers
  - Therefore monad bundles are specified by large numbers of integers
- Aim: Find the MSSM with from Monad Bundle over a CY
- (An aside: Not the “standard embedding” - not so great for phenomenology)

Reinforcement Learning and Monad Bundles

Monads and Non-Abelian Bundles

- Essentially only one known model before this (Anderson et al 2011)
  - On the "bicubic": $\begin{matrix}\mathbb{P}^2 \\ \mathbb{P}^2\end{matrix}\left[\begin{matrix}3 \\ 3\end{matrix}\right]^{2,83}$
  - $0 \to V \to \mathcal{O}_X(1,0)^3 \oplus \mathcal{O}_X(0,1)^3 \to \mathcal{O}_X(1,1) \oplus \mathcal{O}_X(2,2) \to 0$
  - Rank (6,2) monad -> rank 4 bundle -> SU(4) bundle -> SO(10) GUT in 4D

$$\mathbf{248}_{E_8} \to \Big[\underbrace{(\mathbf{1},\mathbf{45})}_{\text{gauge bosons}} \oplus \underbrace{(\mathbf{4},\mathbf{16})}_{\text{families}} \oplus \underbrace{(\overline{\mathbf{4}},\overline{\mathbf{16}})}_{\text{anti-families}} \oplus \underbrace{(\mathbf{6},\mathbf{10})}_{\text{Higgs}} \oplus \underbrace{(\mathbf{15},\mathbf{1})}_{\text{bundle moduli}}\Big]_{SU(4)\times SO(10)}$$

$$n_{\mathbf{16}} = h^1(X,V)\ , \qquad n_{\overline{\mathbf{16}}} = h^1(X,V^\star) = h^2(X,V),$$
$$n_{\mathbf{10}} = h^1(X,\wedge^2 V)\ , \qquad n_{\mathbf{1}} = h^1(X,V\otimes V^\star).$$

Long time to calculate - easier to just check index during training

Background

Computation - Reinforcement Learning (RL) - Schematic

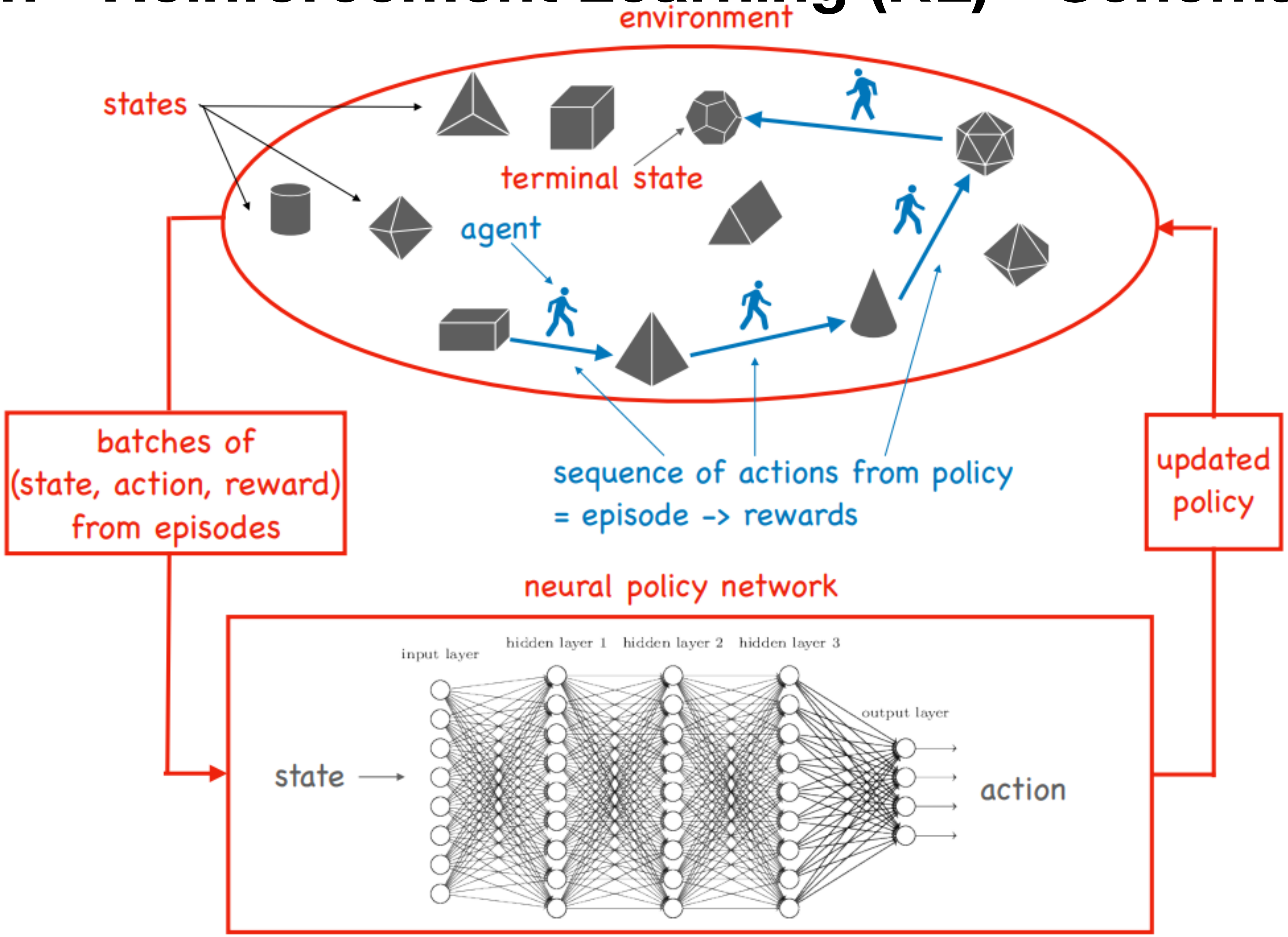


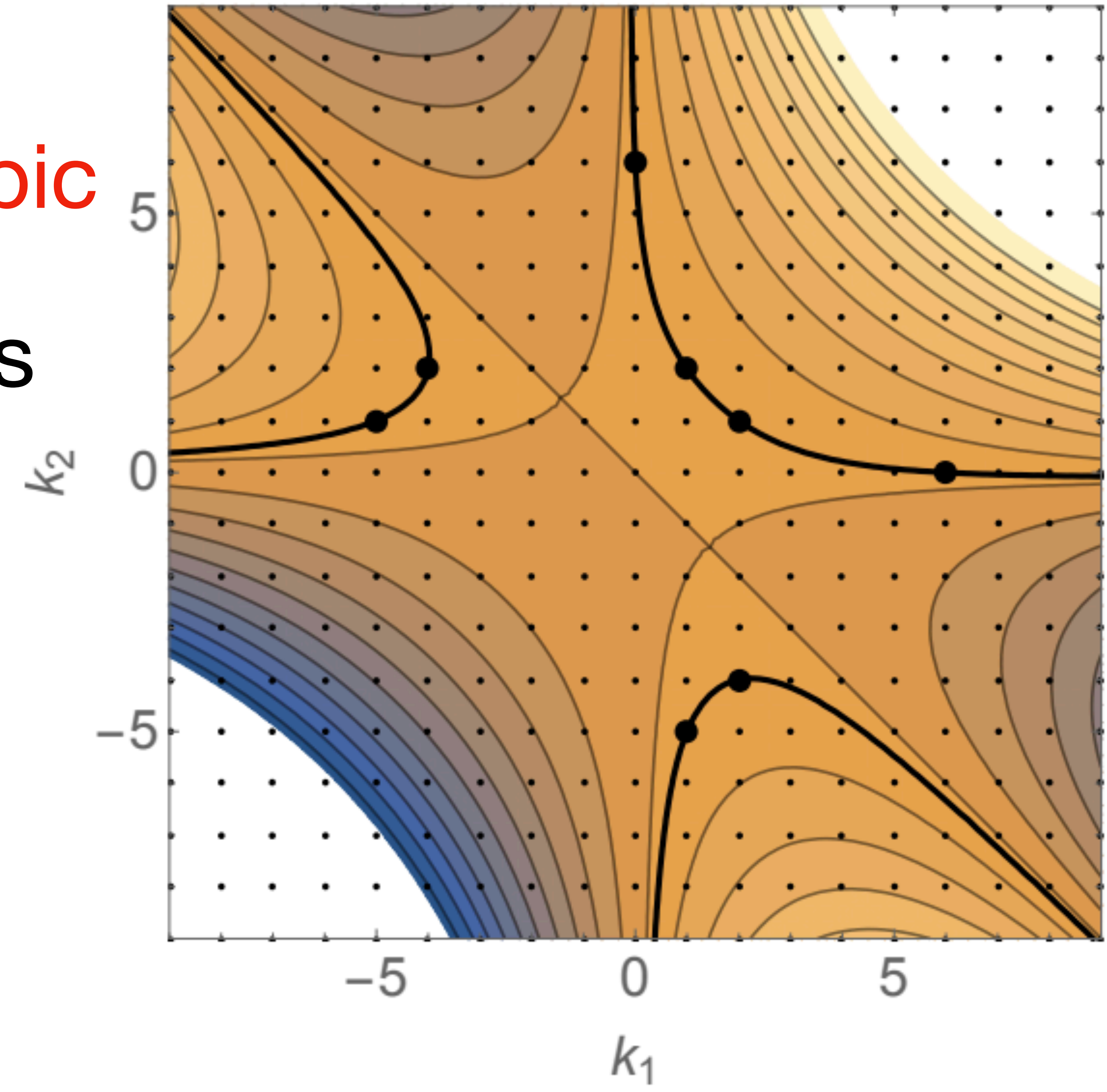
Image Credit: Andre Lukas

Background

Computation - Reinforcement Learning (RL) - Toy Example for Searching

- Find line bundles with with target index τ (=18) on bicubic
  - Solving a cubic Diophantine equation in two variables

$$\frac{1}{6}d_{ijl}k^i k^j k^l + \frac{1}{12}c_{2i}(TX)k^i$$

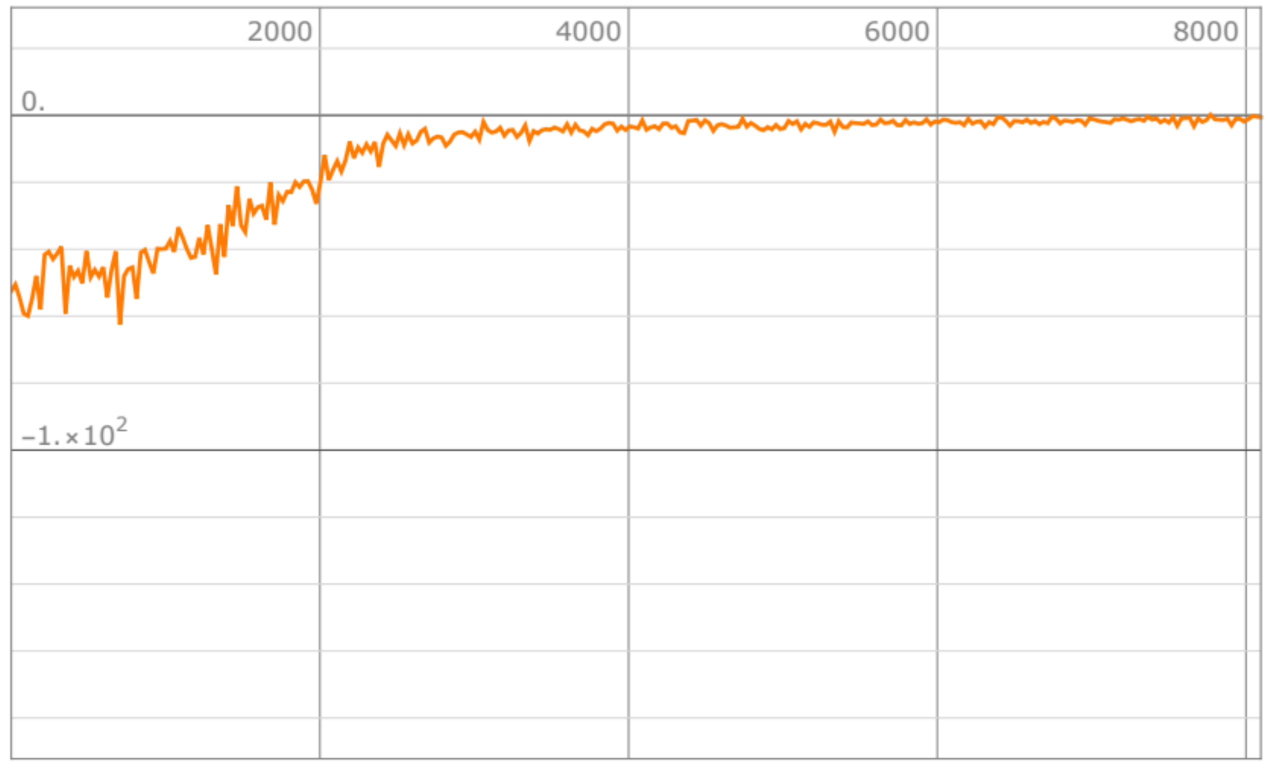


- Environment: Space of line bundles (2D integer lattice)
- Action: Move one space in lattice
- Intrinsic state value: $v(\mathbf{k}) = -\frac{10\,|\mathrm{ind}(\mathcal{O}_X(\mathbf{k})) - \tau|}{h k_{\max}^3}$
- Reward: $r_{s\mapsto s'} = \left\{ \begin{array}{cc} (v(s') - v(s))^p & \text{if} \quad v(s') - v(s) > 0 \\ r_{\text{offset}} & \text{if} \quad v(s') - v(s) \leq 0 \end{array} \right\} + r_{\text{step}} + r_{\text{boundary}} + r_{\text{terminal}}$

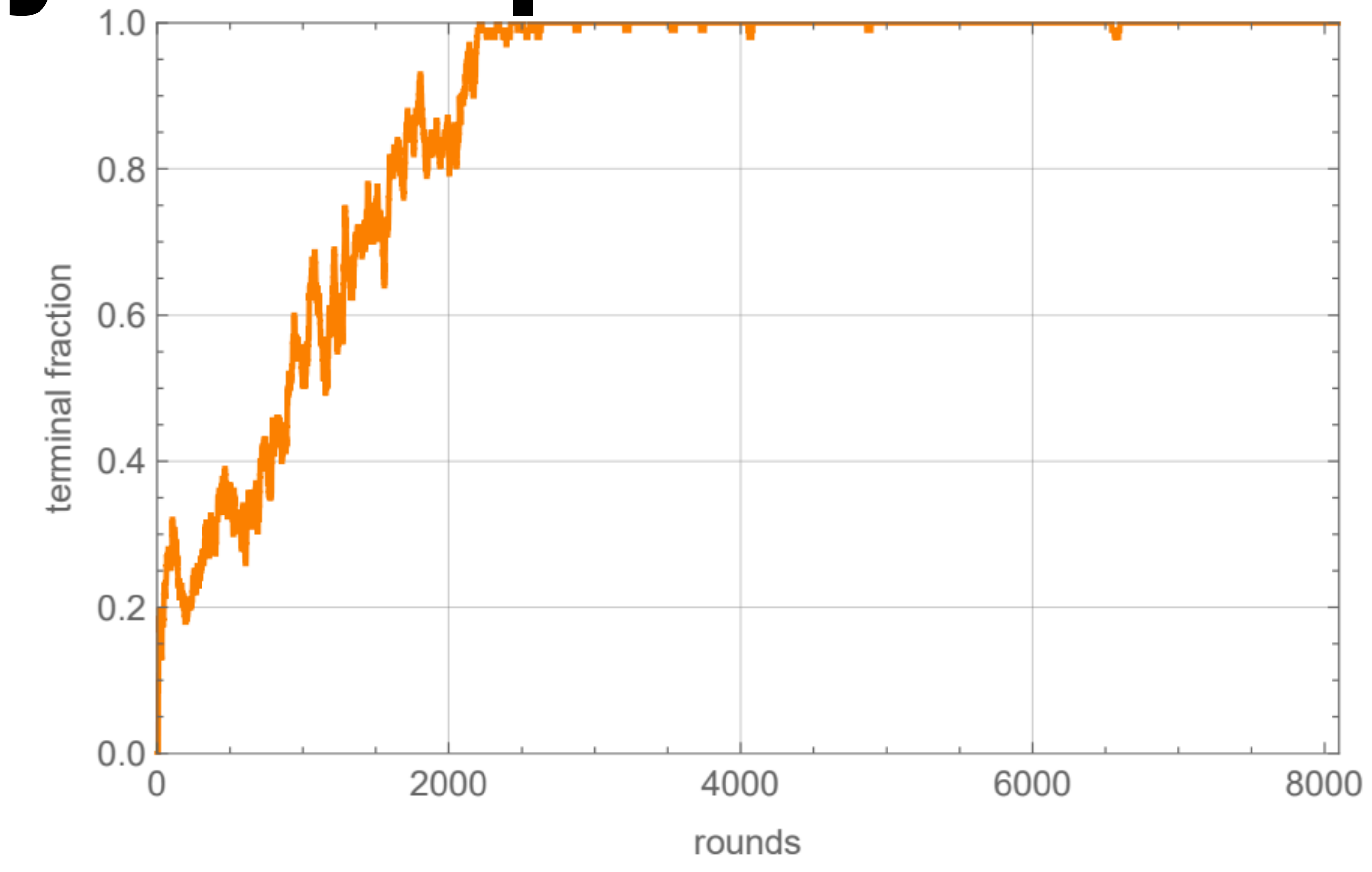
$$p = 1\,, \quad r_{\text{offset}} = -1\,, \quad r_{\text{step}} = 0\,, \quad r_{\text{boundary}} = -1\,, \quad r_{\text{terminal}} = 2\,.$$

Background

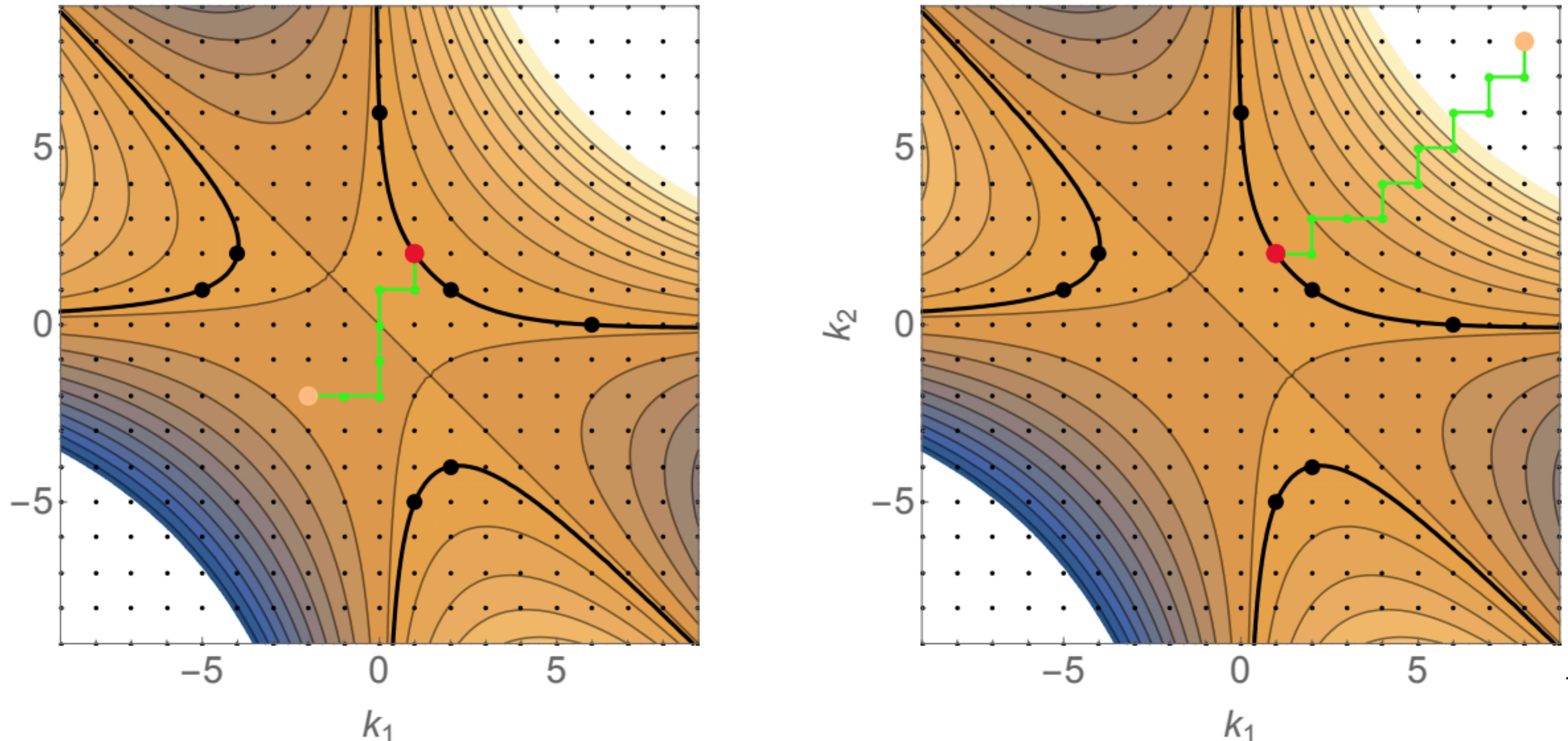
Computation - Reinforcement Learning (RL) - Toy Example

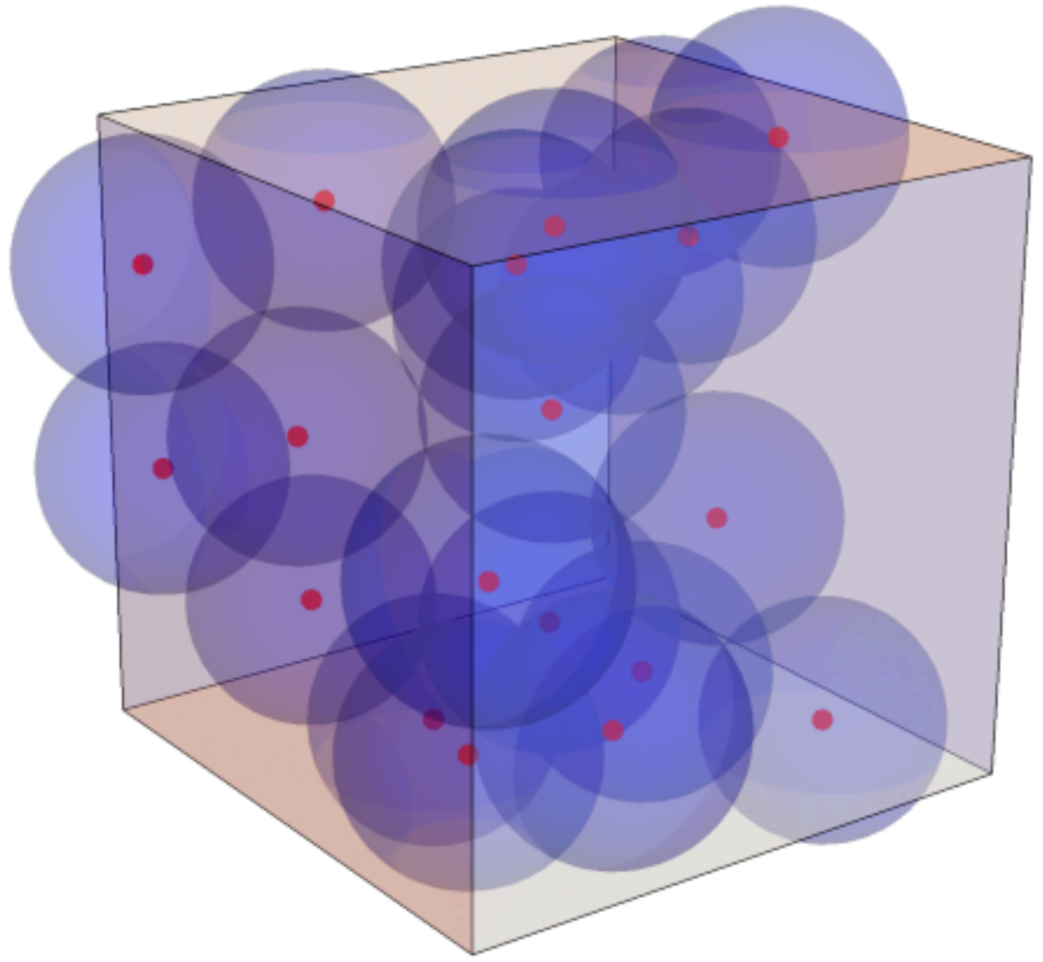


(a) Loss vs batch number.



(b) Fraction of terminal episodes vs episode number.





Reinforcement Learning and Monad Bundles

Encoding for RL

- Environment: monads on given CY3 (large lattice)

$$\mathcal{S} = \left\{ (\mathbf{b}_1, \ldots, \mathbf{b}_{r_B}, \mathbf{c}_1, \ldots, \mathbf{c}_{r_C}) \,|\, b_{\text{min}} \leq b_i^k \leq b_{\text{max}},\ c_{\text{min}} \leq c_a^k \leq c_{\text{max}},\ \sum_{i=1}^{r_B} \mathbf{b}_i = \sum_{a=1}^{r_C} \mathbf{c}_a \right\}$$

- Actions: Move two spaces in the lattice

$$\mathcal{A} = \{\mathbf{b}_i \mapsto \mathbf{b}_i \pm \mathbf{e}_k,\ \mathbf{c}_a \mapsto \mathbf{c}_a \pm \mathbf{e}_k \,|\, i = 1, \ldots, r_b,\ a = 1, \ldots, r_C,\ k = 1, \ldots, h\}$$

- Intrinsic state value: next page
  - How close to MSSM realisation?
- Reward:

$$r_{s \mapsto s'} = \left\{ \begin{array}{cc} (v(s') - v(s))^p & \text{if} \quad v(s') - v(s) > 0 \\ r_{\text{offset}} & \text{if} \quad v(s') - v(s) \leq 0 \end{array} \right\} + r_{\text{step}} + r_{\text{boundary}} + r_{\text{terminal}}$$

$$p = 1.2\ , \quad r_{\text{offset}} = -2\ , \quad r_{\text{step}} = -1\ , \quad r_{\text{boundary}} = -2\ , \quad r_{\text{terminal}} = 10\ .$$

Reinforcement Learning and Monad Bundles

Encoding for RL

Intrinsic state value:

We check cohomology constraints on the terminal states found during training, along with a better stability check

property	term in $v(B, C)$	comment
index match	$-\frac{2\lvert\mathrm{ind}(V)-\tau\rvert}{hM^3}$	$\tau = -3\lvert\Gamma\rvert$ is the target index, $\mathrm{ind}(V)$ computed from Eq. (2.20)
anomaly	$\frac{1}{hM^2}\sum_{i=1}^{h}\min\left(c_{2i}(TX) - c_{2i}(V), 0\right)$	no penalty if anomaly condition satisfied, $c_{2i}(V)$ computed from Eq. (2.20)
bundleness	$-(d_{\mathrm{deg}}+1)$	d_{deg} = dimension of degeneracy locus as discussed in Sec. 2.4; if the degeneracy locus is empty, d_{deg} is to be taken as -1
split bundle	$-n_{\mathrm{split}}$	n_{split} = number of splits in V
equivariance	$-\sum_{U\subset B,C}\mathrm{mod}(\mathrm{ind}(U), \lvert\Gamma\rvert)$	U runs over all line bundles in B, C or blocks of same line bundles, as discussed in Sec. 2.4
trivial bundle	$-n_{\mathrm{trivial}}$	n_{trivial} = number of trivial line bundles
stability V	$-\frac{\max(0, h^0(X,B) - h^0(X,C))}{hM^3}$	tests Hoppe's criterion for V, cohomologies from formulae in Sec. 2.3
stability V^*	$-\frac{\max(0, h^0(X,B^*) - h^0(X,C^*))}{hM^3}$	tests Hoppe's criterion for V^*, cohomologies from formulae in Sec. 2.3

Reinforcement Learning and Monad Bundles

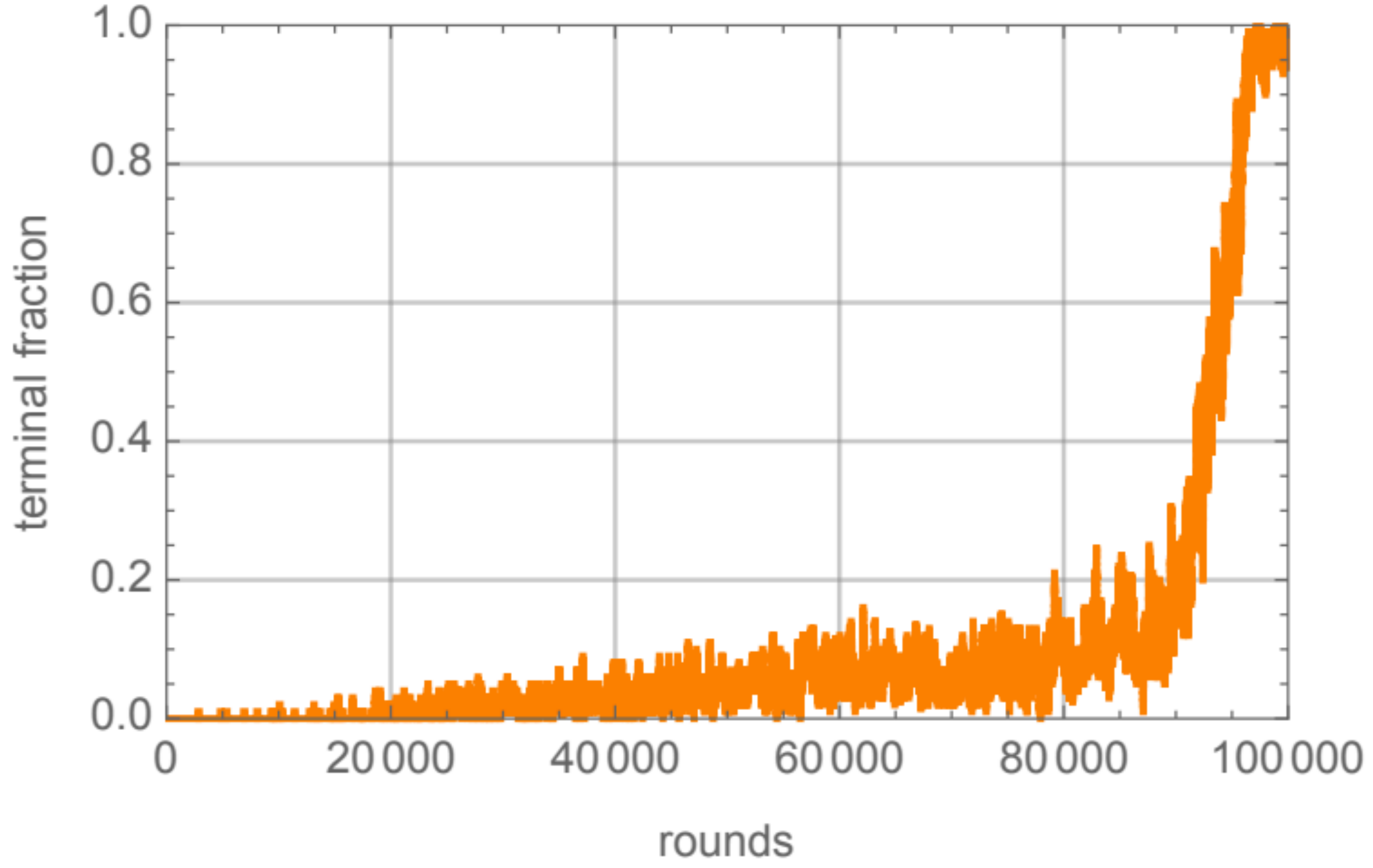
Training

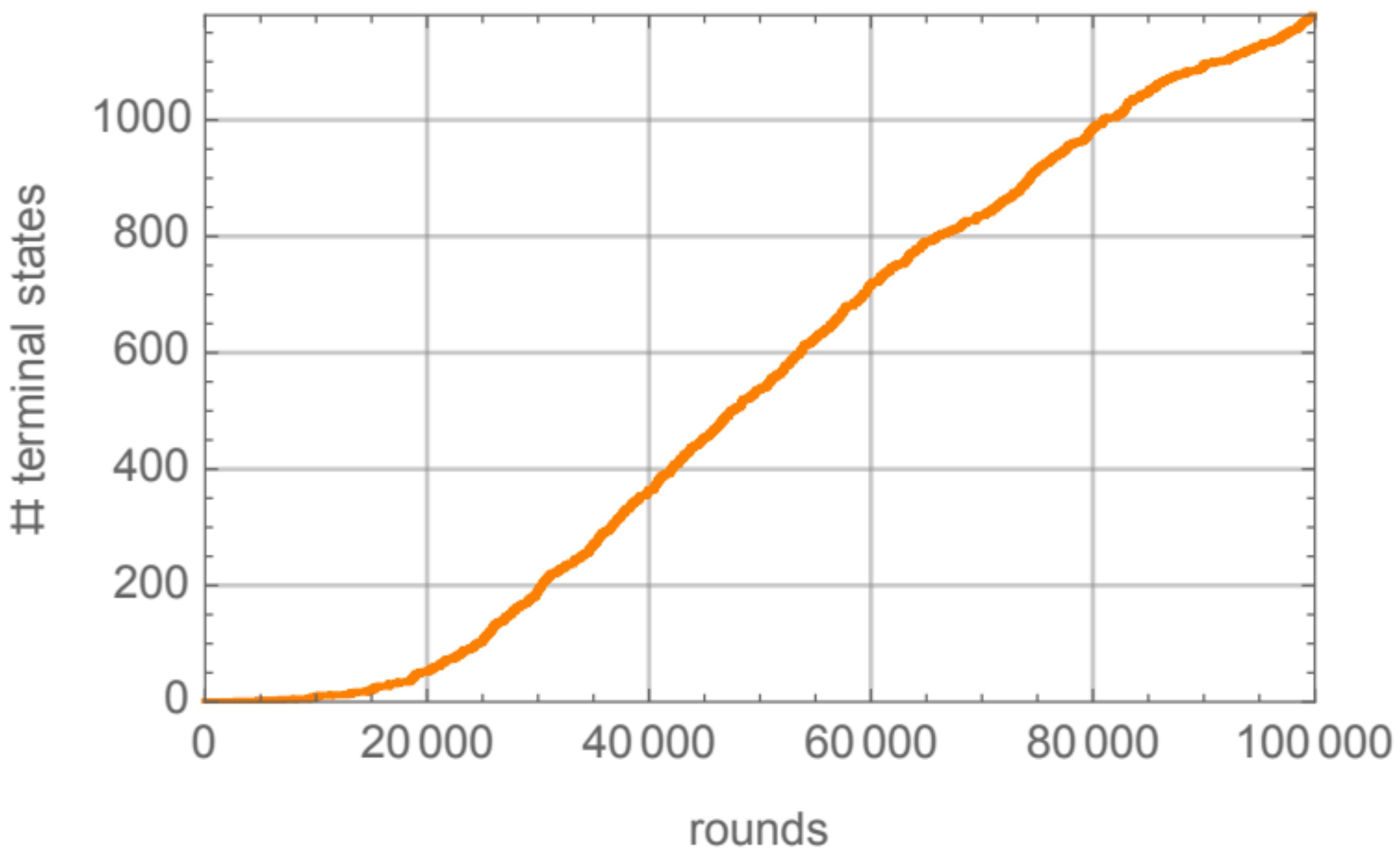
Bicubic - (6, 2) monads - this is the same space as the known model

b = -3…5 & c = 0… 5 -> 10^13 states in total

After removing redundancies (and extra checks), the known model is the only example without negative line bundle entries

After ~1 hour on a single CPU, we find ~15 models (after extra checks)





Reinforcement Learning and Monad Bundles

What is the Network doing?

Appears to solve inequalities first

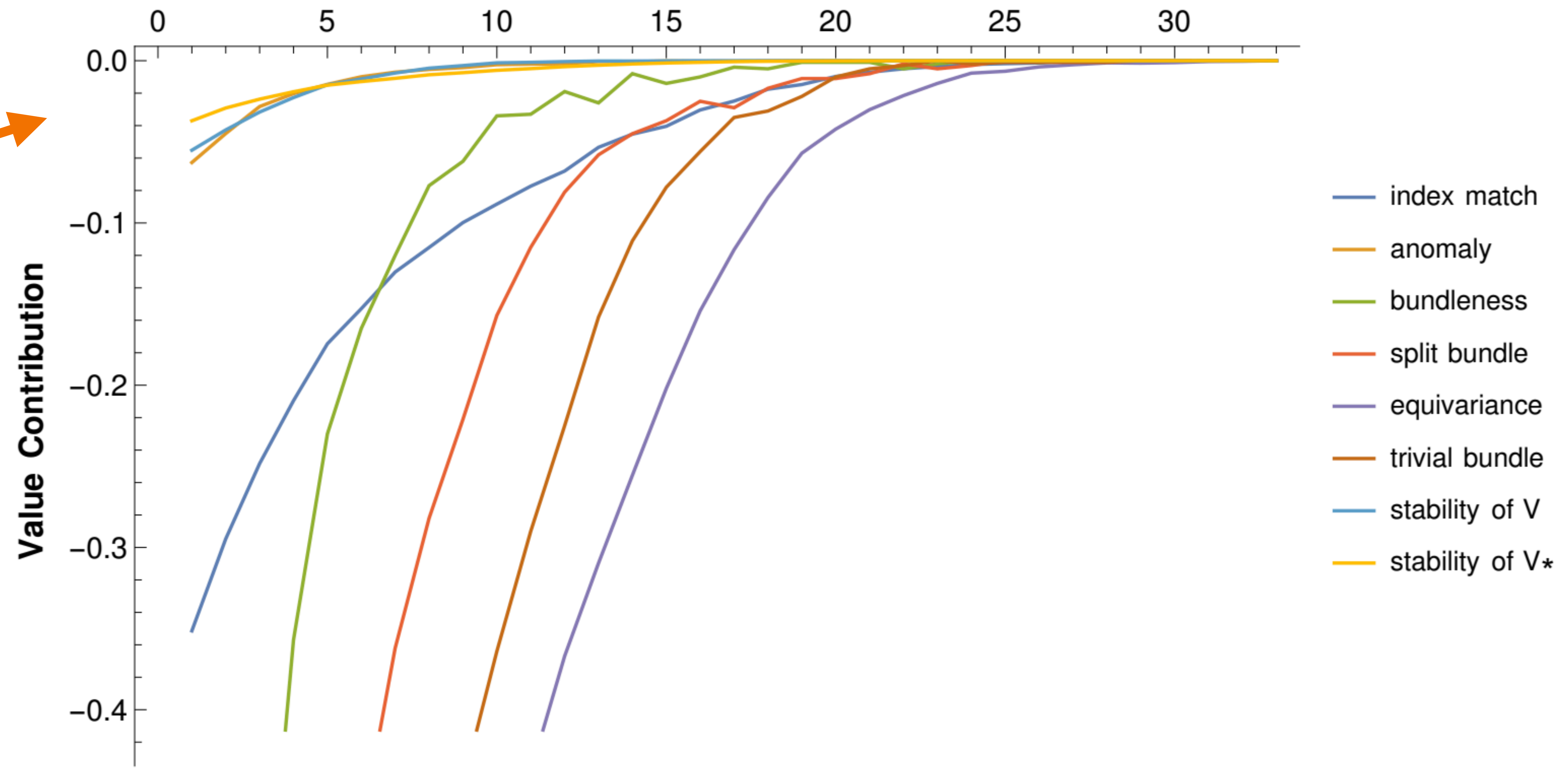
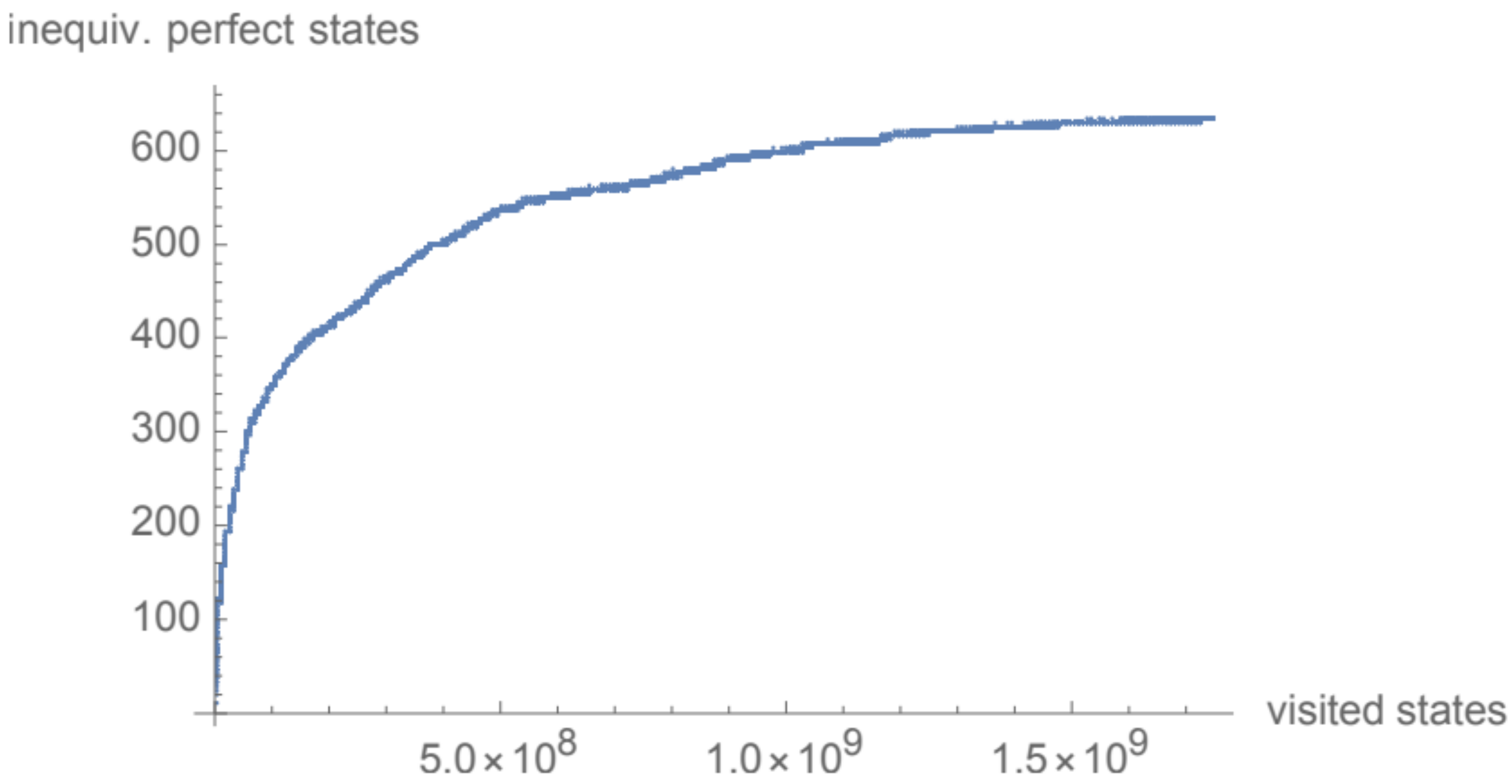


Figure 7: The different contribtutions to the intrinsic value for $(r_b, r_c) = (6, 2)$ bicubic models. This data is averaged over 1000 termianl states using the trained network.

Pushing to Saturation



Also have O(500) models on triple tri-linear

$$X \sim \left(\begin{array}{c|ccc} \mathbb{P}^2 & 1 & 1 & 1 \\ \mathbb{P}^2 & 1 & 1 & 1 \\ \mathbb{P}^2 & 1 & 1 & 1 \end{array}\right)$$

Found after further cohomology checks (35 genuine (6,2) Monads)

Found similar success with genetic algorithms for discrete optimisation

Comparison to searches

We also have found similar success with genetic algorithms and environments up to ~10^28

Manifold	h	$\lvert\Gamma\rvert$	Range	GA	Scan	Found	Explored
7862	4	2	[-7,8]	5	5	100%	10^{-10}
7862	4	4	[-7,8]	30	31	97%	10^{-10}
7447	5	2	[-7,8]	38	38	100%	10^{-14}
7447	5	4	[-7,8]	139	154	90%	10^{-14}
5302	6	2	[-7,8]	403	442	93%	10^{-19}
5302	6	4	[-7,8]	722	897	80%	10^{-19}
4071	7	2	[-3,4]	11,937	N/A	N/A	10^{-14}

Calculating Quark Masses from String Theory

Yukawa Couplings in String Theory

- Couplings require information about geometry (not just topology)
- This involves finding the metric on the Calabi-Yau and other field profiles
- i.e. Solve the 6D Einstein equations, coupled to matter! Very hard
- Have recently managed to do this with these line bundle sums! This also involved machine learning methods, where the metric and fields are represented by NNs, and the loss function is made from the PDEs
- First calculation of this kind - made possible by ML!

Computation of Quark Masses from String Theory

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Why do you need geometry now?

- Fields are not canonically normalised in a string compactifications
- $\mathscr{L} = - K_{i\bar{j}}\bar{\psi}^j\gamma_\mu D^\mu\psi - K_{i\bar{j}}D_\mu\bar{\phi}^j D^\mu\phi - (\lambda_{ijk}\phi^i\psi^j\psi^k + h.c.) + \dots$

Field space metric

Holomorphic Yukawa Couplings

$$K_{IJ} \sim \int_{CY} \nu_I \wedge \star_V \nu_J$$

$$\lambda_{IJ} \sim \int_{CY} \nu_I \wedge \nu_J \wedge \nu_K$$

Hodge Star - Depends on the Metric!

What is v?

$$K_{IJ} \sim \int_{CY} \nu_I \wedge \star_V \nu_J \qquad \lambda_{IJ} \sim \int_{CY} \nu_I \wedge \nu_J \wedge \nu_K$$

$$\triangle_{10}\, \phi(x,y) = (\, \triangle_4 + \triangle_6\,)\phi(x,y) \qquad \phi(x,y) = \sum_I \varphi_I(x)\nu_I(y), \quad \triangle_6\, \nu_I(y) = m_I^2 \nu_I(y)$$

$$\boxed{\triangle_4\, \phi_n(x) + m_n^2 \phi_n(x) = 0}$$

We only want the zero modes

The derivative is the gauge and gravity covariant derivative

Geometry: Metric $\longrightarrow$ Gauge Field $\longrightarrow$ Harmonic forms

The String Model

This model has the **MSSM Particle Content + Uncharged Moduli**

No extra vector-like pairs or chiral exotica

Hypersurface in A = $\mathbb{P}_1 \times \mathbb{P}_1 \times \mathbb{P}_1 \times \mathbb{P}_1 \sim S^2 \times S^2 \times S^2 \times S^2$

SU(5) Like structure but no GUT phase

$$p = \sum_{\text{even}} x_\alpha^2 y_\beta^2 u_\gamma^2 v_\delta^2 + \psi_0 \sum_{\text{odd}} x_\alpha^2 y_\beta^2 u_\gamma^2 v_\delta^2 + \psi\, x_0 x_1 y_0 y_1 u_0 u_1 v_0 v_1$$

$$2\begin{pmatrix} Q_2 \\ U_2 \\ E_2 \end{pmatrix}, \begin{pmatrix} Q_5 \\ U_5 \\ E_5 \end{pmatrix}, \begin{pmatrix} D_{2,4} \\ L_{2,4} \end{pmatrix}, 2\begin{pmatrix} D_{4,5} \\ L_{4,5} \end{pmatrix}, H^d_{2,5}, H^u_{2,5} .$$

$$V = \mathcal{O}_X \begin{array}{c} \begin{array}{ccccc} L_1 & L_2 & L_3 & L_4 & L_5 \end{array} \\ \begin{pmatrix} -1 & -1 & 0 & 1 & 1 \\ 0 & -3 & 1 & 1 & 1 \\ 0 & 2 & -1 & -1 & 0 \\ 1 & 2 & 0 & -1 & -2 \end{pmatrix} \end{array}$$

$$Y_u \sim \begin{bmatrix} 0 & 0 & * \\ 0 & 0 & * \\ * & * & 0 \end{bmatrix} \qquad Y_d \sim \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

First Calculation of its kind! Proof of concept - do not expect realistic results

Equations to Solve

Correction to a reference metric: $g_{CY,a\bar{b}} = g_{FS,a\bar{b}} + \partial_a \partial_{\bar{b}} \phi, \quad \mathscr{L}_{\text{MA}} \sim \left| 1 - \frac{1}{\kappa} \frac{\det g}{\Omega \wedge \bar{\Omega}} \right|_p$

This is called the Monge-Ampere equation.
It is equivalent to the vacuum Einstein equations for our purposes

Correction to a reference connection on each line bundle: $A = H^{-1} \bar{\partial} H$

$H^E = e^{\beta} H^E_{FS}$, solve $\Delta\beta = \rho_\beta, \quad \mathscr{L}_{\text{HYM}} \sim \left| \Delta\beta - \rho_\beta \right|_p$

Correction to a reference 1-form for each field: $\nu = \nu_{ref} + \bar{\partial}_{L_i} \sigma_\theta, \quad \mathscr{L}_{\text{one-form}} \sim \left| \Delta_{L_i} \sigma - \rho_\sigma \right|_p$

ϕ and β are functions while σ is a section. They need to transform appropriately - imposed with architecture

For the up Yukawas, we need 11 NNs in total

(Equivariant) Projective Neural Networks

"A tensor is something that transforms like a tensor!"

$$\mathbb{P}_1 = \frac{\mathbb{C}^2 - \vec{0}}{\mathbb{C}*} \quad \Rightarrow \quad [x, y] = [\lambda x, \lambda y] \; \forall \lambda \in C*$$

Functions: $f(\lambda x, \lambda y) = f(x, y)$

$$\pi_\theta : [x, y] \to \left[\frac{x\bar{y}}{|x|^2 + |y|^2}, \frac{x\bar{x}}{|x|^2 + |y|^2}, \frac{y\bar{y}}{|x|^2 + |y|^2} \right] \to \cdots\text{feed-forward network}\cdots \to \mathbb{R}$$

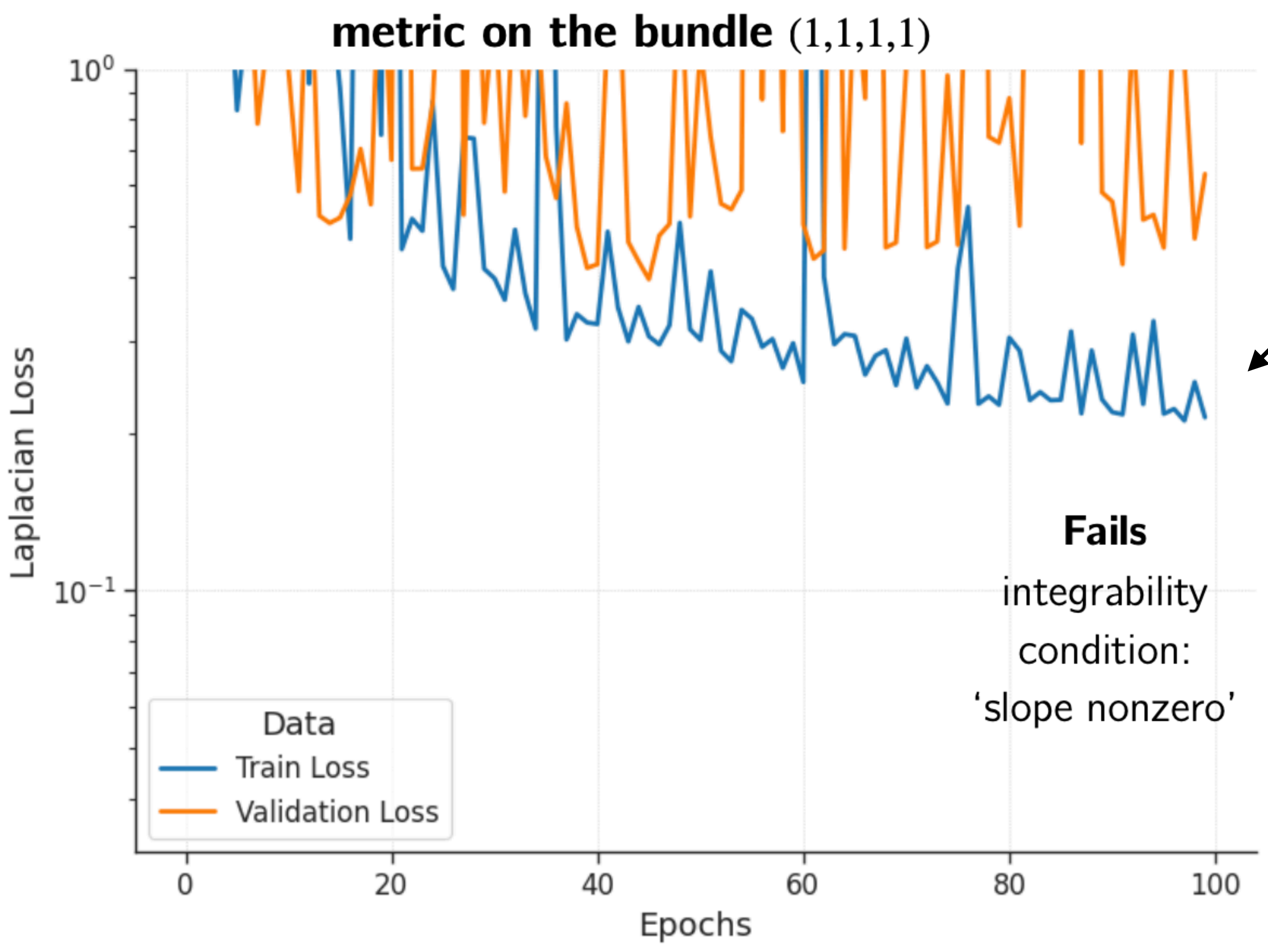
Sections: $\sigma \in \mathcal{O}_{\mathbb{P}_1}(n) \Rightarrow \sigma(\lambda x, \lambda y) = \lambda^n \sigma(x, y)$

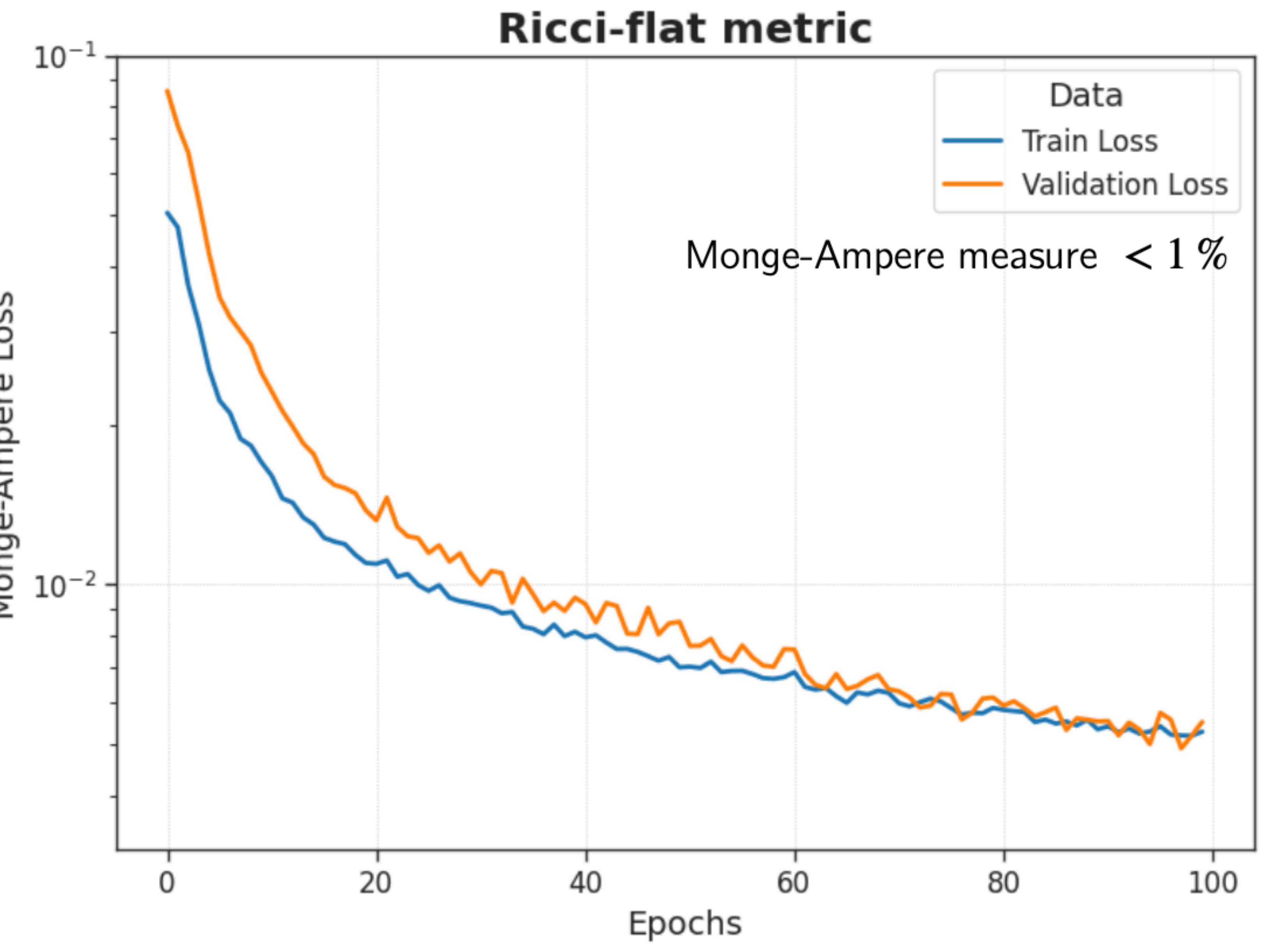
$$n = 2 \quad \pi_\theta : [x, y] \to \left[\frac{x\bar{y}}{|x|^2 + |y|^2}, \frac{x\bar{x}}{|x|^2 + |y|^2}, \frac{y\bar{y}}{|x|^2 + |y|^2} \right] \to \cdots\text{feed-forward network}\cdots \to \mathbb{R}^6 \to (a, b, c) \in C^3 \to ax^2 + bxy + cy^2$$

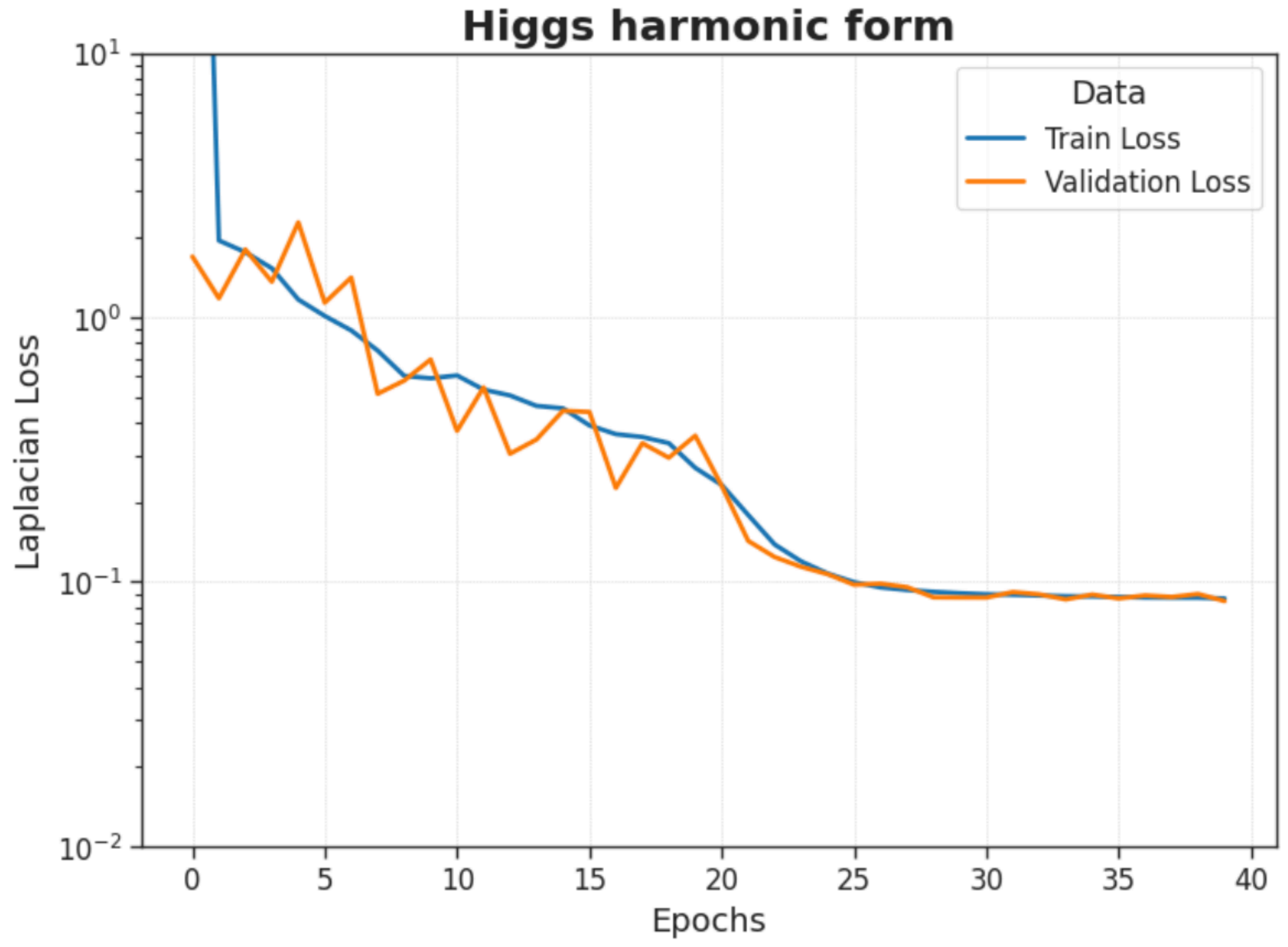
These generalise to P_1^4 and higher dimension projective spaces

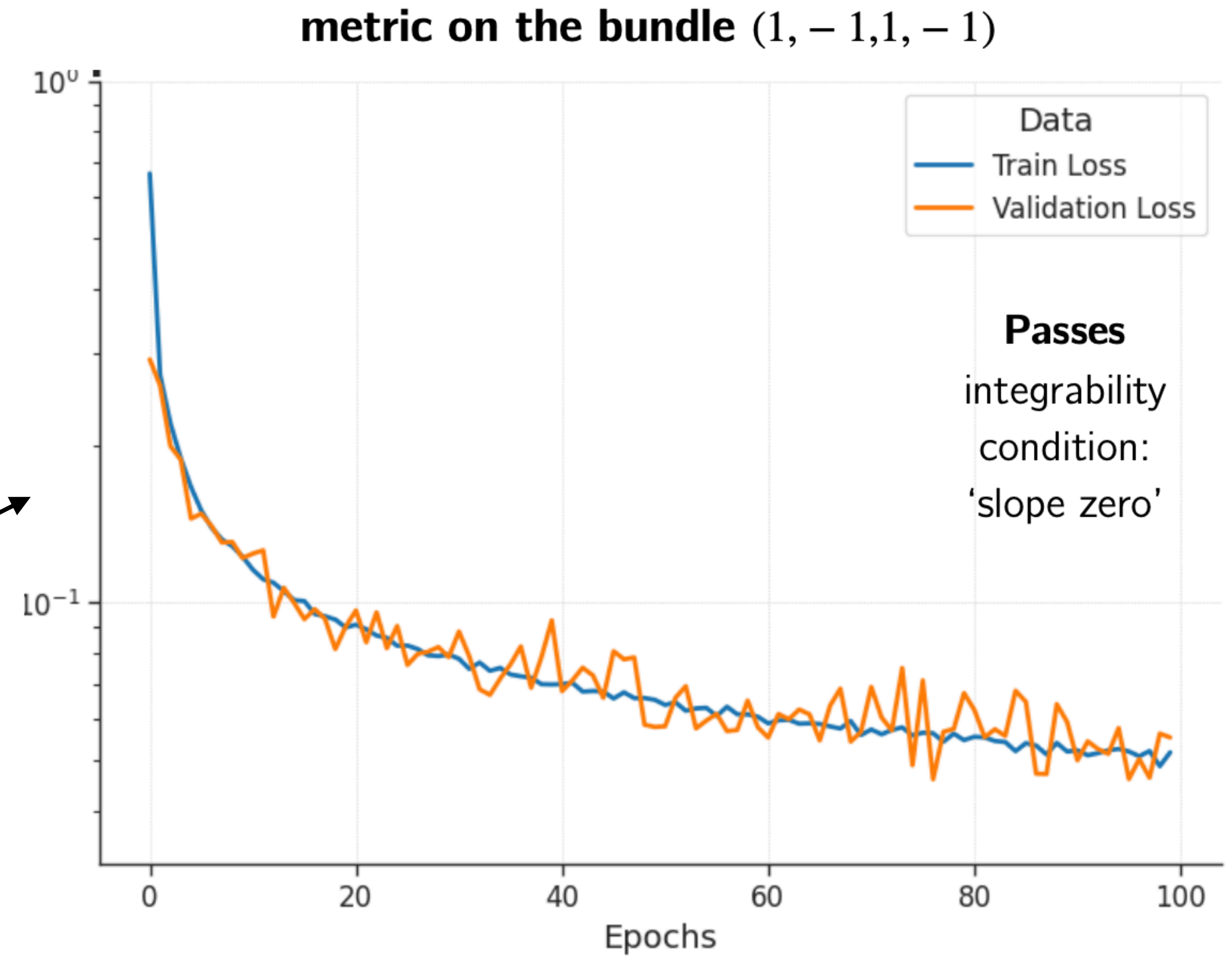
Training

Topological obstruction to solving EOM









All satisfy the EOM to within a few percent
(This is the error on the formal correction)

Topological allowed to solve EOM

Results

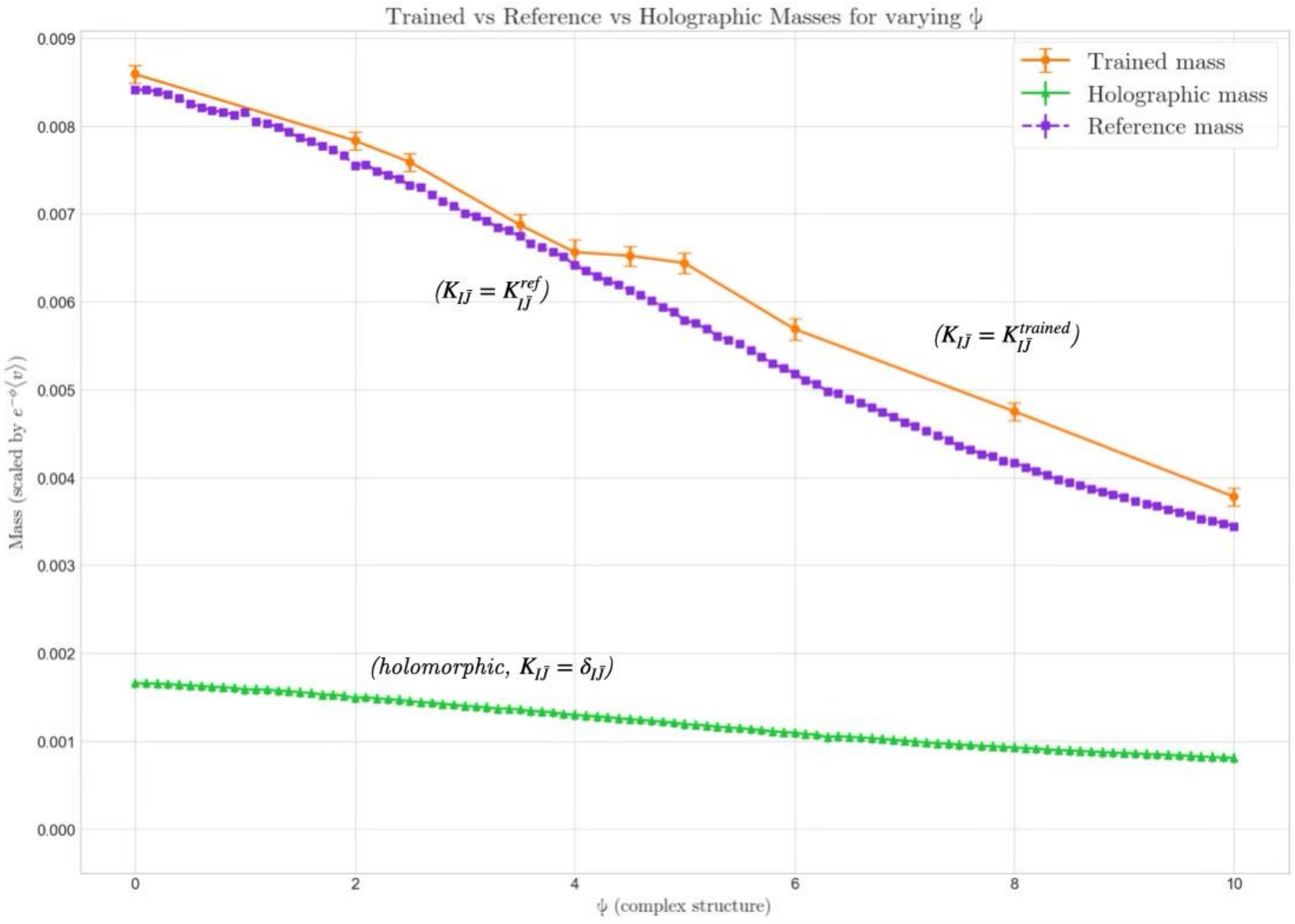
From model structure, will always have one massless quark

From choice of one-parameter family of moduli two remaining masses are equal

We checked that this degeneracy is lifted for other choices of moduli

Statistically ~1% error

Reference quantities are unexpectedly very close (~10% error)



Conclusions

- Methods from data science allow for searches for interesting string vacua
- RL can be used to Engineer string vacua with specific properties
  - In our case to lead to the the MSSM
  - Starts finding examples after exploring 10^{-7} of the environment
  - These methods can be applied more broadly to large landscapes in physics
- Solve Einstein's equations and Yang-Mills equations in 6D
  - NNs represent solutions
  - Can use this to calculate previously inaccessible quantities